Proceedings of the
10th CONGRESS OF
YUGOSLAV MATHEMATICIANS
Belgrade, 21–24.01.2001, 337–341

Section:
Computer sciences

COMPARING PERFORMANCES OF SEVERAL ALGORITHMS FOR SOLVING THE SIMPLE PLANT LOCATION PROBLEM

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Abstract. In literature there exist several survey articles about the Simple Plant Location Problem (SPLP), which describe classical (dual-based) algorithms and all of their properties. In this paper several new techniques are described for solving the SPLP with comparative analysis of their results in practice. Results are also compared to dual based algorithms as best classical methods.

1. Problem formulation

The simple plant location problem (SPLP) is a well-known NP-hard combinatorial optimization problem ([9]) and basic member in family of location problems. The problem is also known as uncapacitated warehouse location problem or uncapacitated facility location problem.

Consider:

- a set $I = \{1, \dots, m\}$ of candidate sites for facility location, and
- a set $J = \{1, \dots, n\}$ of customers.

Each facility $i \in I$ has a fixed cost f_i. Every customer $j \in J$ has a demand b_j, and c_{ij} is the unit transportation cost from facility i to customer j.

It has to be decided:

- facilities to be established and
- quantities to be supplied from facility i to customer j

such that the total cost (including fixed and variable costs) is minimized.

Mathematically, the SPLP is formulated by (1)–(3):

$$\min\Big(\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} + \sum_{i=1}^{m} f_i y_i\Big) \tag{1}$$

subject to:

$$\sum_{i=1}^{m} x_{ij} = 1, \qquad \text{for } j \in J; \tag{2}$$

$$0 \leqslant x_{ij} \leqslant y_i \text{ and } y_i \in \{0,1\}, \qquad \text{for } i \in I \text{ and } j \in J; \tag{3}$$

AMS Subject Classification: 90 C 27, 90 C 35, 90–08

where: x_{ij} represents the quantity supplied from facility i to customer j, y_i indicates whether facility i is established ($y_i = 1$) or not ($y_i = 0$).

2. Methods for solving SPLP

2.1. Classical algorithms.

The presentation of all important contributions relevant to SPLP lies beyond the scope of this paper. Some important survey articles are [5] and [9]. We are going to mention only several efficient and well-known classical methods for solving SPLP.

The DUALOC algorithm by Erlenkotter [2] has been the fastest algorithm for solving SPLP for a long time. This algorithm is based on a linear programming dual formation (LP dual) in condensed form that evolved in simple ascent and adjustment procedure. If ascent and adjustment procedures do not find optimal solution, Branch-and-Bound (BnB) procedure completes the solution process. Using this algorithm it is possible to solve hard SPLP instances up to 500 facility sites and customers.

Körkel in [8] showed how to modify a primal-dual version of Erlenkotter's exact algorithm to get an improved procedure. The computational experience with large-scale problem instances indicated that speedup to DUALOC is significant (more than one order of magnitude). By this approach is solved SPLP instances over the 3000 facility sites and customers. Unfortunately, this commercial product has been practically unavailable to us. We only can to adopt conclusion in paper [8], which says that PDLOC generates solutions 10-100 times faster than DUALOC.

2.2. Evolutionary approaches.

The first simple GA method for solving SPLP was proposed in [10] which solving ORLIB instances ([1]) in reasonably short time. It is significantly improved using a better objective value function, by caching GA and Add-heuristic. Further improvements of GA operators produce implementation that is capable for efficiently solving large-scale SPLP instances over the 2000 facilities and customers ([11] and [12]).

In that approach the population size is 150 individuals and population is randomly initialized in first generation. Binary representation of facility sites and efficient objective value function is used. Rank based selection is performed with rank 2.5 for best individual down to 0.7 for worst, by step 0.012. This selection scheme successfully prevents premature convergence in local optima and loosing the genetic material. Uniform crossover and simple mutation is used as other genetic operators, with steady-state generation replacement with elitist strategy. In each generation only 1/3 of population is replaced, with 2/3 of population that is directly passed into the next generation. In [4] rank based selection scheme (RB) is compared with fine grained tournament selection (FGT) proposed in [3]. Inconsiderable better results are obtained by using FGT.

Other GA method was proposed in [6] and tested on ORLIB instances. It applies a clustering technique as mutation guidance and a novel local search method to enhance the solution quality of the basic simple GA scheme. A binary chromosome representation is used and population is randomly initialized in first generation with population size of 20 (with local search) or 50 individuals (without local search). Applied crossover rate (one-point crossover) was 0.95, mutation rate 0.05 and maximal number of generations was 300. Simple mutation operator is applied with 1/3 probability and clustering technique guided with 2/3 probability. Clustering divide the facilities into the two groups according to their fixed costs. One contains facilities of a higher fixed cost and the other contains those of lower costs. If the transportation costs is substantially lower than fixed costs some customers are reassigned from the lower cost group into the higher cost group.

2.3. Other methods.

Lagrangean relaxation and subgradient optimization is directly applied in [7] as an attempt to upgrade the standard DUALOC algorithm. This procedure slightly sharpening penalties and using alternative rules for node selection and the selection of the branching variable in the BnB procedure. But these theoretical improvements give very degraded performance in practice, as can be seen in Table 2.

In [13] is also applied similar approach incorporated into the Lagrangean heuristic for solving SPLP and Capacitated Facility Location Problem (CFLP). This heuristic accelerates a classical subgradient method, using the local information of the surrogate constraints relaxed in the Lagrangean relaxation. Several computational tests in that paper confirm the superiority of this scheme compared to classical Lagrangean heuristic.

Tabu Neural Network (TANN) in [14] integrated an analog version of the short term memory component of tabu search with neural networks to generate a massively parallel, analog global search strategy. TANN was applied on SPLP instances of small dimensions (10×10, 20×20 and 30×15). Obtained solutions have error more than 20% from optimal ones, which means that TANN is completely useless in practice.

3. Performance comparison

In this section results of previously described approaches are summarized. Instances of equal size are grouped and average value is reported. The columns in Table 1 and 2 describe:

- Names of SPLP instances;
- Dimensions m and n of instances in that group;
- Results obtained from [6] on Pentium PC/166MHz represented by:
  - number of optimally solved instances,
  - overall runs and
  - running time in seconds;
- Results from [12] on AMD 80486/133MHz represented on same way as previous (with RB selection scheme);

- Running times of RB and FGT selection schemes respectively ([4]) on Pentium III/600MHz;
- DUALOC BnB ([2]) running times in seconds on AMD 80486/133MHz. Note that DUALOC found optimal solutions and proved that they are optimal for instances 41-MQ and 3 instances in MR group;
- DUALOC without branching (only Dual ascent + Dual adjustment procedures) on AMD 80486/133MHz;
- Performances of algorithm given in [7] in seconds on 80486/33MHz (all solutions are optimal);
- Results of Lagrangean heuristic ([13]) on IBM 6000 3AT workstation.

Instance	Dimension	Horng Pent. 166	Kratica RB 486/133	Fil. RB PIII/600	Fil. FGT PIII/600
41-74	16×50	200/200 2.883	260/260 0.86	0.10	0.05
81-104	25×50	180/200 4.48	240/240 1.26	0.17	0.18
111-134	50×50	173/200 8.296	198/240 2.97	0.51	0.55
A-C	100×1000	83/150 443.61	42/60 83.1	16.28	12.24
MO	100×100	not tested	93/100 5.49	0.59	0.35
MP	200×200	not tested	100/100 16.60	1.18	0.51
MQ	300×300	not tested	100/100 34.97	2.69	0.93
MR	500×500	not tested	99/100 93.69	4.68	2.52
MS	1000×1000	not tested	100/100 379.6	23.18	14.47
MT	2000×2000	not tested	100/100 1812	not tested	not tested

Table 1. Results of evolutionary approaches

Inst.	DUALOC Opt 486/133	DUALOC Heur 486/133	Klose Opt 486/33	Lorena IBM 6000 3AT
41-74	<0.01	13/13 <0.01	0.135	not tested
81-104	<0.01	12/12 <0.01	0.308	not tested
111-134	<0.01	12/12 0.013	1.201	not tested
A-C	25.17	error 5.74%	15 891	2/3 32.61
MO	32.54	error 9.74%	not tested	not tested
MP	369.7	error 9.63%	not tested	not tested
MQ	2913	error 10.4%	not tested	not tested
MR	3/5 >50 000	error 13.35%	not tested	not tested
MS	break - error 11.32%	error 15.29%	not tested	not tested
MT	not tested	not tested	not tested	not tested

Table 2. Results of other methods

4. Conclusion

Comparative results given in previous section shows superiority of dual method DUALOC on small size ORLIB instances. Note that it always produces an optimal solution and verified its optimality. In class of evolutionary approaches, our GA gives better quality of solutions more than 10 times faster than Horng GA, estimated for equal CPU, as can be seen from Table 1.

In the case of medium and large scale instances MO-MT our GA implementation is quite superior to other approaches (produces best results with minimal running time). Speedup to DUALOC is more than 500 times for MR instances and we believe that speedup for instances MS and MT increases exponentially with instance size.

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