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A Survey of Genetic Algorithm Approaches for Solving Hub Location Problems

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Abstract Hub facilities are used as consolidation, connection and switching points between two locations in transportation and telecommunication networks. The hub location problem is concerned with locating hub facilities and allocating demand nodes to hubs in order to route the traffic between origin-destination pairs. By employing hub nodes as switching points in the network and increasing transportation between them, the capacity of network can be used more efficiently. In this paper we survey and compare different genetic algorithm (GA) approaches for solving several NP-hard hub location problems. We also include some recent trends on hub location models from the literature.

Keywords Hub Location · Genetic Algorithms · Discrete Optimization · Network Design

1 Introduction

Hubs arise in the design of telecommunications networks, airline passenger networks and postal delivery networks. These networks involve a set of interacting facilities that exchange some form of communication among each other. This communication interaction could be in forms such as airline passengers, data packets or mail flows. Hubs are special facilities that serve as switching, routing and sorting points in transportation and telecommunication networks. Instead of serving each origindestination pair

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directly, hub facilities concentrate flows in order to take advantage of economies of scale.

The hub location problem is concerned with locating hub facilities and allocating demand facilities to hubs in order to route the traffic between origindestination pairs. The goal is to concentrate flows of freight or passengers in order to exploit the economies of scale in transportation. The design of a hub network is a key determinant of the cost and competitiveness of a transportation and logistics system.

One important difference between classical facility location and hub location problems is that two allocation schemes exist in hub networks. Regarding the way terminal nodes are assigned to hubs, we have single allocation and multiple allocation variants of hub location problems. In multiple allocation problems, the traffic of a same terminal can be routed through different hubs. On the contrary, in single allocation problems, each terminal is assigned to a single hub.

There may be capacity restrictions on the volume of traffic a hub can collect and/or receive, capacity restrictions on demand nodes or arcs in the network, etc. These problems are named capacitated hub location problems. In hub location problems with fixed costs, a fixed costs are be associated with establishing hub nodes and/or hub arcs. If the number of hubs is fixed, say equal to p, these problems are named as p-hub problems in the literature.

In hub location problems there is a given network with n nodes on which the set of origins, destinations and potential hub locations are identified. The flow in the network (representing cost, time, distance,...) is known. Hub location problems usually involve the following assumptions:

(a) the hub network is complete with a link between every hub pair,
(b) there is economies of scale incorporated by a discount factor α for using the inter-hub connections,
(c) direct communication between two non-hub nodes is not allowed.

Under the assumptions mentioned above, the goal in hub location problems is to locate the set of hubs and to allocate the demand nodes so that the total cost of the network, or the worst origin-destination cost is minimized.

The hub location problems have received a lot of attention in the literature in the past two decades. Hub location models are often derived from practice and most of them are NP-hard (see [21] and [15]). Hub location researchers have developed various exact, heuristic and hybrid methods for solving hub location problems. One review of hub location problems and their classification can be found in [10] and [11],.

The scope of this paper is to give a brief survey of genetic algorithm approaches (GAs) that have been successfully applied for solving eight classical hub location problems:

(1) The Uncapacitated Multiple Allocaton p-hub Median Problem UMApHMP,
(2) The Uncapacitated Multiple Allocaton p-hub Center Problem - UMApHCP,
(3) The Uncapacitated Single Allocation p-Hub Median Problem - USApHMP,
(4) The Uncapacitated Single Allocation p-Hub Center Problem - USApHCP,
(5) The Capacitated Single Allocation p-Hub Median Problem - CSApHMP,
(6) The Capacitated Single Allocation p-Hub Center Problem - CSApHCP,
(7) The Capacitated Single Allocation Hub Location Problem - CSAHLP,

Computational characteristics of the proposed approaches are evaluated through extensive computational experiments using two benchmark test instances: the Civil

Aeronautics Board (CAB) and Australia Post (AP) data sets. Computational performance of the proposed approach was compared with the optimal/best solutions found in the literature.

The rest of this paper is divided into five sections. Mathematical models of the considered hub location problems are in Section 2. Basic characteristics of the GA methods applied to hub location problems are given in Section 3. Comments on the computational results are reported in Section 4. Conclusions and suggestions for future research are discussed in the last section.

2 Mathematical models

2.1 Hub median models

The research on hub location began with the work of OKelly [41], [42] and [40], although Goldman [22] is the first paper addressing the network hub location problem. Most of hub location research has been devoted to hub median problems, in which the main goal is to design a network with hubs in order to minimize the total transportation cost and possibly the costs of establishing such a network.

One of the first hub median problems that was studied in the literature is the Uncapacitated Multiple Allocation p-hub Median Problem UMApHMP. The problem was formulated as a linear integer program in [8]. Several improvements of this formulation arise in the literature: [48], [16] and [5]. The mixed integer linear programming formulation proposed in [5] is used in this paper. This formulation uses the following notation:

$I = 1, ..., n$ = set of of n distinct nodes in the network where each node corresponds to origin/destination or potential hub location,
C_{ij} = the distance from node i to node j,
W_{ij} = the demand from an origin node i to a destination node j,
p=the number of hubs to be located,
χ, δ =unit costs for collection (origin-hub) and distribution (hub-destination),
α = a discount factor for the hub-hub transportation.

Decision variables H_j, Z_{ik}, ${Y_{kl}}^i$ and ${X_{lj}}^i$ are used in the formulation as follows:

$H_j = 1$, if a hub is located at node j, 0 otherwise
Z_{ik} = the amount of flow from node i that is collected at hub k
${Y_{kl}}^i$ = the amount of flow from node i that is collected at hub k, and transported via hub l
${X_{lj}}^i$ = the amount of flow from node i to destination j that is distributed via hub l

Using the notation mentioned above, the UMApHMP can be written as:

$$\min \sum_i [\chi \sum_k C_{ik} Z_{ik} + \alpha \sum_k \sum_l C_{kl} {Y_{kl}}^i + \delta \sum_l \sum_j C_{lj} {X_{lj}}^i] \tag{1}$$

subject to:

$$\sum_i H_i = p \tag{2}$$

$$\sum_k Z_{ik} = \sum_j W_{ij} \quad \text{for every } i \tag{3}$$

$$\sum_l X^i_{lj} = W_{ij} \quad \text{for every } i, j \tag{4}$$

$$\sum_l Y^i_{kl} + \sum_j X^i_{kj} - \sum_l Y^i_{lk} - Z_{ik} = 0 \quad \text{for every } i, k \tag{5}$$

$$Z_{ik} \leq \sum_j W_{ij} H_k \quad \text{for every } i, k \tag{6}$$

$$\sum_i X^i_{lj} \leq \sum_i W_{ij} H_l \quad \text{for every } l, j \tag{7}$$

$$X^i_{lj}, Y^i_{kl}, Z_{ik} \geq 0, H_k \in \{0, 1\} \quad \text{for every } i, j, k, l \tag{8}$$

The objective function (1) minimizes the sum of the origin-hub, hub-hub and hub-destination flow costs multiplied with parameters χ, α and δ respectively. Constraint (2) limits the number of located hubs to p, while (3)-(5) represent the divergence equations for the network flow problem for each node i. Constraints (6) and (7) prevent direct communication between non-hub nodes, while (8) reflects non-negative and/or binary representation of decision variables.

The Uncapacitated Single Allocation p-hub Median problem USApHMP is similar to the UMApHMP, but each non-hub node may be allocated to a single node. This can be formulated similarly, but we can restrict the Z_{ik} variables to be binary and eliminate the ${X_{lj}}^i$ variables. The binary H_k variables for locating hubs are replaced with $H_{ij} \in \{0, 1\}$. Let H_{ij} have value 1 if node i is allocated to a hub node j and 0 otherwise. The condition $H_{kk} = 1$ implies that the node k is a hub.

Using the notation mentioned above, in [40] the problem can be written as :

$$min \sum_{i,j,k,l \in I} W_{ij}(\chi C_{ik} H_{ik} + \alpha C_{kl} H_{ik} H_{jl} + \delta C_{jl} H_{jl}) \tag{1}$$

subject to:

$$\sum_{k=1}^{n} H_{kk} = p \tag{2}$$

$$\sum_{k=1}^{n} H_{ik} = 1 \quad for\ every\ i = 1, ..., n \tag{3}$$

$$H_{ik} \leq H_{kk} \quad for\ every\ i, k = 1, ..., n \tag{4}$$

$$H_{ik} \in \{0, 1\} \quad for\ every\ i, k = 1, ..., n \tag{5}$$

The constraint (3) guarantees a single hub allocation for each node. By the constraint (5) we prevent hub nodes being allocated to other nodes. The constraint (4) enforces that the flow is sent only via open hubs, thus preventing direct transmission between non-hub nodes.

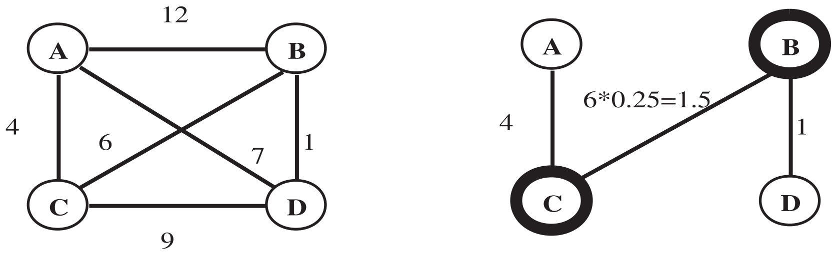

Fig. 1 Hub network n=4, p=2 for the USApHMP

On the left side of Fig.1 one example of the network with n=4 nodes A, B, C, D and the distances between them is given. Solving the USApHMP with parameters p=2, $\chi = \delta = 1$ and α=0.25, we obtain the optimal solution presented on the right side of Fig. 1. As it can be seen two hub nodes are located at B and C, while non-hub nodes A and D are associated to hubs C and B respectively. The transportation costs between every pair of nodes are: ”A-*C*-A”=4+4=8, ”A-*C*-*B*”= 4+6*0.25=5.5, ”A-*C*”=4, ”A-*C*-*B*-D”=4+6*0.25+1=6.5, ”*B*-*B*”=0, ”*B*-*C*” = 6*0.25=1.5, ”*B*-D” = 1, ”*C*-*C*”=0, ”*C*-*B*-D” = 6*0.25+1=2.5 and ”D-*B*-D” =1+1=2. The objective value (total transportation cost) is equal to 52. Note that the non-hub nodes in the paths are denoted in italic letters.

The Capacitated Single Allocation Hub Location Problem -CSAHLP involves single allocation scheme and limited amount of flow collected at each hub. The number of hubs to be located is not predetermined. The fixed costs for locating hubs are also assumed. Let:

Γ_k = the collection capacity of hub k,
F_k = the costs of establishing hub k,
O_i = the amount of flow that departs from node i; i.e. $O_i = \sum_{j=1}^{n} W_{ij}$ and
D_j = the amount of flow that is distributed to node j, i.e. $D_j = \sum_{i=1}^{n} W_{ij}$.

Using the notation mentioned above, in [17] the CSAHLP is formulated as:

$$\min \sum_{i=1}^{n}\sum_{k=1}^{n} C_{ik} Z_{ik}(\chi O_i + \delta D_i) \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} \alpha C_{kl} Y_{kl}^{i} + \sum_{k=1}^{n} F_k Z_{kk} \tag{1}$$

with constraints:

$$\sum_{k=1}^{n} Z_{ik} = 1 \quad \text{for every } i = 1, ..., n \tag{2}$$

$$Z_{ik} \leq Z_{kk} \quad \text{for every } i, k = 1, ..., n \tag{3}$$

$$\sum_{j=1}^{n} W_{ij} Z_{jk} + \sum_{l=1}^{n} Y_{kl}^{i} = \sum_{l=1}^{n} Y_{lk}^{i} + O_i Z_{ik} \quad \text{for every } i, k = 1, ..., n \tag{4}$$

$$\sum_{i=1}^{n} O_i Z_{ik} \leq \Gamma_k Z_{kk}; \quad \text{for every } k = 1, ..., n \tag{5}$$

$$Y_{kl}^{i} \geq 0 \quad \text{for every } i,k,l = 1,...,n \tag{6}$$

$$Z_{ik} \in \{0,1\} \quad \text{for every } i,k = 1,...,n. \tag{7}$$

The objective function (1) minimizes the sum of transportation cost between all origin-destination pairs via hub nodes and the fixed costs of locating the set of hubs. The amount of flow that is collected in a hub is limited by (5).

The Capacitated Single Allocation p-Hub Median Problem CSApHMP involves assumptions as the CSAHLP, with two differences:the number of hubs is predetermined to p and there are no fixed costs for establishing hubs. The formulation of the CSApHMP is obtained from the formulation of the CSAHLP, by omitting the part $\sum_{k=1}^{n} F_k Z_{kk}$ from the objective function and by adding the constraint $\sum_{k=1}^{n} Z_{kk} = 1$, fixing the number of located hubs to p.

2.2 Hub center models

The p-hub median formulation can sometimes lead to unsatisfactory results, for example, when the worst origin-destination distance (cost) is important. This can happen, for example, in designing fast delivery systems, where delivery time is depends on the worst-case distance between origin and destination node. In these cases, p-hub center models are more appropriate. The objective function is computed by minimizing maximum delivery time of packages. Since the delivery time in the worst case is computed, it is obvious that in other cases required delivery time interval will also be satisfied. Two earliest definitions of p-hub center problem can be found in [43] and [9].

The formulation of the Uncapacitated Multiple Allocation p-hub Center Problem-UMApHCP uses the same notation and assumptions as the UMApHMP. A free variable z represents the objective, while parameters $\chi = \delta = 1$. The UMApHCP problem can be written as:

$$min \quad z \tag{1}$$

with the constraints:

$$\sum_{k=1}^{n} H_k \quad = \quad p \tag{2}$$

$$X_{ij}^{k} \leq H_k \quad for \;\; every \;\; i,j,k \in I \tag{3}$$

$$Y_{ij}^{l} \leq H_l \quad for \;\; every \;\; i,j,l \in I \tag{4}$$

$$\sum_{k=1}^{n} X_{ij}^{k} \quad = \quad 1 \quad for \;\; every \;\; i,j \in I \tag{5}$$

$$\sum_{l=1}^{n} Y_{ij}^{l} \quad = \quad 1 \quad for \;\; every \;\; i,j \in I \tag{6}$$

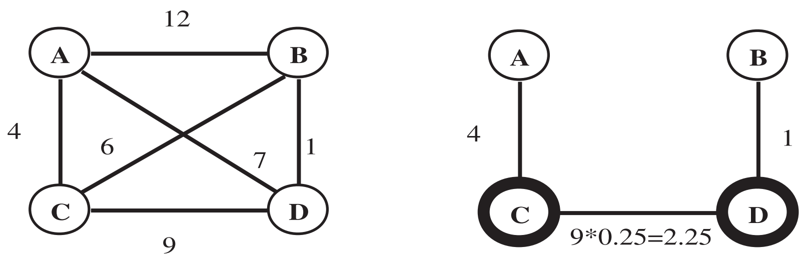

Fig. 2 Hub network n=4, p=2 for the USApHCP

$$z \geq \sum_{k=1}^{n} (C_{ik} + \alpha \cdot C_{kl}) \cdot X_{ij}^{k} + \sum_{m=1}^{n} C_{mj} \cdot Y_{ij}^{m} - \alpha \cdot (1 - Y_{ij}^{l}) \cdot C_{max} \quad for \;\; every \;\; i, j, l \in I \tag{7}$$

$$H_k, X_{ij}^{k}, Y_{ij}^{l} \in \{0, 1\} \quad for \;\; every \;\; i, j, k, l \in I \tag{8}$$

The objective of the UMApHCP is to minimize the maximum of the total flow cost between any pair of nodes in the network. Constraint (7) denotes the lower bound for the objective variable z, which represents the maximum transportation cost between all origin-destination pairs.

The Uncapacitated Single Allocation p-hub Center Problem-USApHCP has the same constraints as the USApHMP, and the notation used for formulating the USApHMP remains. A free variable z is introduced and $\chi = \delta = 1$. In [9], the USApHCP is fomulated as:

$$\min \max_{i,j,k,m \in I} (d_{ik} + \alpha C_{km} + C_{mj}) H_{ik} H_{jm} \tag{1}$$

subject to:

$$\sum_{k \in I} H_{ik} = 1 \quad for \; every \; i \in I \tag{2}$$

$$H_{ik} \leq H_{kk} \quad for \; every \; i, k \in I \tag{3}$$

$$\sum_{k \in I} H_{kk} = p \tag{4}$$

$$H_{ik} \in \{0, 1\} \quad for \; every \; i, k \in I \tag{5}$$

On the Fig. 2 the same network from the previous example from Fig. 1 is considered with the same parameter values. As it can be seen form the right side of the Fig. 2, in the optimal solution of the USApHCP, the hubs are now located at nodes C and D, while non-hub nodes A and B are associated to hubs C and D respectively. The minimal distances between every pair of nodes are: "A-*C*-A"=4+4=8, "A-*C*-*D*-B"=

4+9*0.25+1=7.25, "A-C"=4, "A-C-D"=4+9*0.25+1=6.25, "B-D-B"=1+1=2, "B-D-C" = 1+9*0.25=3.25, "B-D" = 1, "C-C"= 0, "C-D" = 9*0.25=2.25 and "D-D"= 0. The objective value is their maximum: path "A-C-A"=4+4=8.

The Capacitated Single Allocation p-hub Center Problem-CSApHCP assumes capacity restrictions on incoming flow in a hub, single allocation scheme and fixed number of hubs. The problem was first formulated by [51]. Decision variables are $Z_{ij} = 1$ if node i is allocated to a hub node j and $Z_{ij} = 10$ otherwise. The condition $Z_{kk} = 1$ implies that the node k is a hub. A free variable z is also introduced. The CSApHCP can be formulated as:

$$\min z \tag{1}$$

with constraints:

$$\sum_{k=1}^{n} Z_{kk} = p \tag{2}$$

$$\sum_{k=1}^{n} Z_{ik} = 1 \quad \text{for every } i = 1, ..., n \tag{3}$$

$$Z_{ik} \leq Z_{kk} \quad \text{for every } i, k = 1, ..., n \tag{4}$$

$$\sum_{i=1}^{n} O_i Z_{ik} \leq \Gamma_k Z_{kk}; \quad \text{for every } k = 1, ..., n \tag{5}$$

$$z \geq \sum_{k=1}^{n} (\chi C_{ik} + \alpha C_{kl}) + \delta C_{lj} Z_{jl}; \quad \text{for every } i, j, l = 1, ..., n \tag{6}$$

$$Z_{ik} \in \{0, 1\} \quad \text{for every } i, k = 1, ..., n. \tag{7}$$

All seven hub location problems mentioned above are proved to be NP-hard in the literature. Even some of their subproblems remain NP-hard. For example, the subproblem of the USApHCP obtained by fixing the set of hubs, named the Hub Center Single Allocation Problem (HCSAP), is also NP-hard [20]. If there is no cost reduction in transport between the hubs, i.e $\alpha = 0$, we also obtain a NP-hard subproblem of the USApHCP [19]. The Ucapacitated Single Allocation Hub Location Problem-USAHLP (the uncapacitated variant of the CSAHLP) is proved to be NP-hard in [24].

2.3 Recent trends

According to number of publications on hub location problems, it is evident that the interest in the hub location area was and still is very strong. Hub location models have evolved from the classical, first generation models, and newer models are expected to be even more realistic and useful in applied settings. The new, second generation models tend to be more realistic by integrating hub location and network design decisions. Some of the second generation hub location models are given below:

- The hub models with flows capacities and minimum flows on inter-hub links and flow-dependent costs in all network links are described in [7]. The models with non-linear cost-functions are also proposed in [23], [58] and [27].
- Hub networks may involve arcs with discounted cost rates, named hub arcs, see [39], [12], [13] and [14]. The goal is to locate a fixed number of hub arcs in order to minimize the overall cost.
- In [46], [47], [1], [49] and [50], the authors considered a flow threshold model where they do not locate hubs but they decide on the links with reduced unit transportation costs. In their model, the cost of flow is reduced according to a prescribed discount factor, if the flow through that link is larger than a given threshold value
- Hub models with flow-based discounts are described in [6] and [44]. O'Kelly and Bryan [45] proposed a non-linear cost-function which allows cost to increase at a decreasing rate as flows increase.
- The latest arrival hub location problems impose a maximum travel time constraint and then minimize the cost of setting up hub network. These models are introduced by Kara and Tansel [25] and have significant application in designing cargo delivery systems [25], [26], [57], [18] and [59].
- In hub location models with competition [37], the demand node capture from competitor hubs is sought whenever the location of a new hub results in the reduction of origin-destination transportation cost. These models arise from air passenger and cargo transportation networks.

3 Proposed genetic algorithms

Hub location problems are significantly harder to solve than the classical facility location problems or some other well-studied problems as the quadratic assignment problem. Despite the similarity of the constraints of the USApHMP and the well known p-median problem, these two location problems have important differences. The demands in the USApHMP are origin-destination flows and not simply demands from a particular node. Moreover, the objective function of the USApHMP is quadratic in assignment variables. Finally, it may not be optimal to assign node to a nearest hub from the chosen set of hubs. if we concentrate only on the location of the hubs and then simply use the "closest hub" assignment rule, without observing interactions between the nodes, the USApHMP turns into a p-median problem. As has been mentioned above, even if the set of hubs is fixed, the problem of determining the optimum single allocation of demand nodes to hubs (HCSAP) is already NP-hard multi-processor assignment problem, related to the well known NP-hard quadratic assignment problem. If we remove the capacity constraints in the capacitated hub location problems, in most cases the obtained subproblems are still NP-hard (CSApHMP turns to USApHMP, CSApHCP to USApHMP, CMApHMP to UMApHMP, CSAHLP to USAHLP, etc..)

Although hub location problems are similar by names, it turns that the slight modification constraints can significantly change problem structure. For example, the USApHCP and UMApHCP problems have the same objective function and constraints, except the constraint that determines allocation scheme. Difference in only one constraint results in significant change of the computational behavior of the problem. If the set of hub is fixed, the USApHCP turns to NP-hard subproblem HCSAP [20], while UMApHCP reduces to a subproblem that can be solved in polynomial time [11]

The exception are hub location problems with fixed costs, which can be solved by multiple solving the p-median problem ($p = 1, 2, 3, .., n$) and by adding the fixed costs. However, regarding the running times, it turns that this approach is not the best solution method. For other hub location models, the optimal solutions are significantly different, so that the optimal solution of one hub location model can not be used for solving the others.

For the reasons mentioned above, there is no general solution approach for solving all hub problems, or at least a smaller group of them. Exact methods can not provide solutions for large-scale hub location problems, which arise from practice, in a reasonable amount of time. Local search usually usually unsatisfactory results, so that there are not so many heuristic methods in the literature for solving these problems.

Genetic algorithms, as robust heuristic methods (see [2], [3]) are very promising approaches for solving hub location problems. GA is a generic method that can be applied to any problem if the feasible solutions of the problem can be represented as strings that correspond to genetic encoding of the solutions. The GA technique is designed to imitate the selective breeding of organisms in the context of problem solving. This technique has been successfully adapted and applied to many problems in the operations research literature: [29], [36], [56], [53], [34], [35], [38].

In this section, we review the GA approaches proposed for solving seven classical hub location problems mentioned in Sect. 2.1 and discuss their effectiveness on CAB and AP hub data set. We point out the main aspects of the proposed GA methods and the key ideas that were used to adapt the GAs to the hub problems under consideration. The implementation of new encoding schemes, modified genetic operators and other GA aspects, such as adequate generation replacement policy, caching technique, hybridization with other menthods, enabled GAs to produce high-quality solutions even or real size problems. The detailed description of all proposed evolutionary approaches is out of the scope of this paper. More information about these methods can be found in: [32], [33], [51], [52], [54]and [55].

3.1 Representation

- Individuals in the GA implementations for the UMApHMP and UMApHCP, are represented genetic code is a binary string of length n. One in the genetic code denotes that the current node is chosen as hub, zero if not. For example, from the genetic code 000100110111 for $n = 12$ and $p = 6$ we can understand that nodes $4, 7, 8, 10, 11$ and 12 are chosen as hubs.
- In the GAs for solving the USApHCP, USApHMP, CSApHCP, CSApHMP and CSAHLP one or both of the following encoding schemes are used.
  1. Genetic code consists of 2 segments. The first segment is a string of length p where the digits (genes) within a string correspond to the indexes of open hub nodes. The digits in the first string take values from the set $0, 1...., n-1$, according to the numeration of nodes in the network. The second segment in genetic code has exactly $n - p$ genes. Each gene corresponds to one non-hub node and contains the index of located hub, which is assigned to it. For example, genetic code $114|20$ for $n = 5, p = 3$ dentotes than nodes 1,2 and 4 are chosen as hubs, and 2, 4 show the allocations.
  2. Genetic code of an individual consists of n genes, each referring to one network node. First bit in each gene takes value 1 if the current node is located hub, 0

if not. Considering these bit values the array of opened hub facilities is formed. Remaining bits of the gene are referring to the hub that is assigned to the current node. If the current node is a hub, it is assigned to itself and the remaining bits of the corresponding gene are ignored. For each non-hub node, the obtained array of opened hubs is arranged in non-decreasing order of their distances from the particular node. Objective value is then calculated in the same way as in GA1 implementation.
For example, in the genetic code |02|10|10|00|10| first bits in every gene (0, 1, 1, 0, 1) denote established hubs (1, 2 and 4), while remaining parts of genes (2, 0, 0, 0, 0) show assignments.

For each non-hub node, the array of established hubs is created and arranged in non-decreasing order of their distances from that node. This strategy, named nearest neighbour ordering, ensures that closer hubs have higher priority than distant ones in assigning them to non-hub nodes. Sorting the array of established hubs is performed $n - p$ times for each individual in each GA generation, which takes $O((n - p) * p * logp)$ operations. This strategy ensures significant improvement of GA solutions, while the total GA running time is slightly longer. It was implemented in GAs for solving USApHCP, USApHMP, CSApHCP, CSApHMP and CSAHLP.

3.2 Objective function evaluation

Regarding the applied encoding, different schemes for objective function evaluation are used.

- UMApHMP and UMApHCP: the indices of established hubs are obtained from the genetic code. When the set of hubs is fixed, the problems of allocating non-hub nodes to hubs reduce to solving n^2 shortest paths problems, which can be done in $O(n^2p)$ time,
- USApHMP: the nearest neighbour ordering strategy is applied with avoiding the duplication of established hub indices,
- USApHCP: the nearest neighbour ordering strategy, duplication of established hub indices and local improvement strategy are applied,
- CSAHLP: "closest" hub with sufficient capacity scheme is used,
- CSApHCP, CSApHMP: "closest" hub with sufficient capacity scheme, local search improvement with preserving the feasibility and avoiding duplication of established hub indices are implemented.
For hub-median problems, the objective value is then simply evaluated by summing distances origin-hub, hub-hub and hub-destination multiplied with flows and parameters χ, α and δ and by adding fixed costs for established hubs eventually. For hub-center problems the objective function value is the worst origin-hub-hub-destination cost.
If a non-hub node is allocated to a hub with insufficient capacity, the next hub from the array of established hubs is taken .If the number of hubs is different from p we add or delete necessary number of ones from the genetic code.
In the case of capacitated hub location problems, it may happen that a non-hub node is allocated to a hub whose remaining capacity is not enough to satisfy the node's demand. In this case, the next hub from the array of established hubs for

```
parent1:  001100110101  --->  001100110101  --->
 parent2: 011110100001        011110100001
                             ->j       i<-

011100110001  --->  011100110001 --->  011101100001 offspring1
001101000101        001101000101       001100010101 offspring2
 j       i             ->j i<-              j i
```

Fig. 3 Modified crossover operator

the current node that satisfies the capacity constraint is taken. If there is no such a hub, we consider the individual infeasible by setting its fitness to 0.
By using the strategy described above, infeasible individuals are corrected to be feasible in the initial population. The implemented genetic operators are constructed so that the feasibility of the individuals is preserved, which is very important aspect for GA performance. If all the infeasible individuals were encountered in the population, they may become dominant in the following generations and GA might provide no solution or finish in a local optimum.

3.3 Selection

Proposed GAs use fine grained tournament selection operator FGTS, that is an improvement of standard tournament selection. FGTS showed to be successful in cases when it is desirable that the average size of tournament group F_{tour} has rational instead of integer values. Running time for FGTS operator is $O(N_{nonel} * F_{tour})$, but in practice F_{tour} is considered to be constant, that gives $O(1)$ time complexity for FGTS. The value of the parameter F_{tour} is experimentally determinated and set to 5.4 in GAs implementations. The FGTS is the only common operator in the proposed GA implementations. The crossover and mutation operators are adopted to hub problem under consideration, regarding the applied encoding schemes.

3.4 Crossover

- Modified crossover operator is implemented for solving UMApHMP and UMApHCP. (see Fig.3). The operator is simultaneously tracing genetic codes of the parents from right to left, searching the position i on which the first parent has 1 and second 0. The individuals exchange genes on the found position (identified as crossover point), and similar process is performed starting from the left side of genetic code. Operator is searching the position j where the first parent has 0 and the second 1. Genes are exchanged on the j-th position, and the number of located hubs in both individuals is unchanged. Described process is repeated until $j \geq i$ (see *Fig.3*).
  The crossover is performed with the rate $p_{cross} = 0.85$. It means that around 85% pairs of individuals take part in producing offspring.
- Depending on the applied encoding scheme, one of the two crossover operators was applied for solving USApHCP, USApHMP, CSApHCP and CSApHMP.
  1. Double one-point crossover:

```
parent1: | 0 1 1 3 | 2 0 0 1 1 0 |  ---> | 0 1 1 3 | 2 0 0 1 1 0 |
parent2  | 2 2 5 9 | 0 2 2 1 1 0 |       | 2 2 5 9 | 0 2 2 1 0 0 |
               i           j
---> | 0 1 5 9 | 2 0 0 1 0 0 |  offspring1
     | 2 2 1 3 | 0 2 2 1 1 0 |  offspring2
           i           j
```

Fig. 4 Double one-point crossover operator

```
parent1 |02|01|10|00|10|10|00| ---> |02|01|10|00|10|10|00| --->
parent2 |01|10|00|10|01|01|10|      |01|10|00|10|01|01|10|
                                       i          j
--->  |02|10|10|00|10|01|00| --->
      |01|01|00|10|01|10|10|
                i  j
---> |02|10|10|10|01|01|00| offspring1
     |01|01|00|00|10|10|10| offspring2
```

Fig. 5 Modified one-point crossover

```
 parent1|02|10|01|10|10| ---> |02|10|10|00|10|
 parent2|01|10|10|00|01|      |01|10|01|10|01|
              i
  ---> |02|10|01|10|10|  --->
       |01|10|10|00|01|
                i
    --->|02|10|01|00|01| offspring1
        |01|10|10|10|10| offspring2
```

Fig. 6 Modified one-point crossover for the CSAHLP

In each segment of parents' genetic codes a crossover point is randomly chosen and genes are exchanged after chosen position (Fig. 4). Crossover is performed with the rate $p_{cross} = 0.85$.
2. Modified one-point crossover
The operator is simultaneously tracing genetic codes of the parents from right to left, searching the gene i where $parent1$ has one and $parent2$ zero on the first bit position of this gene(see Fig. 5). The individuals then exchange whole genes on the found position. Similar process is simultaneously performed starting from the left side of parents' genetic codes. Operator is searching the gene j where the $parent1$ has one and $parent2$ zero on the first bit position. These genes are exchanged and the number of located hubs in both individuals is unchanged. The described process is repeated until $j > i$.
3. In the GA implementation for solving the CSAHLP, another variant of modified one-point crossover is applied (Fig. 6) A bit position i is randomly chosen in the genetic code (crossover point). Whole genes are exchanged after the chosen crossover point.Crossover is also performed with the rate p_{cross}=0.85.

```
gen.code 1: 0110010110
gen.code 2: 1100010110
gen.code 3: 0111000110
gen.code 4: 0110010101
gen.code 5: 0111000110
frozen    :  F  F FF
```

Fig. 7 Frozen bits in the genetic code for n=10, p=5

```
|02|01|10|00|10|02|02|10|
|02|01|10|00|10|10|02|00|
|12|01|02|01|10|10|01|01|
|02|10|00|00|10|10|02|00|
|02|01|10|00|11|00|01|10|
|02|01|10|00|10|10|02|00|
  F       F  F     F
```

Fig. 8 Frozen bits in the genetic code for n=8

3.5 Mutation

- In the UMApHMP, UMApHCP modified simple mutation with frozen bits is implemented. Mutation operator is performed by changing a randomly selected gene in the genetic code (0 to 1, 1 to 0) with certain mutation rate.
  During GA run, it may happen that all individuals have the same bit value in a certain position in a gene (frozen bits). The appearance of frozen genes may increase the possibility of premature convergence significantly (see Fig. 7).
  Basic mutation level is 0.4/n for non-frozen and 1.0/n for frozen bits (2.5 higher mutation level for frozen bits).
  Mutation operator counts and compares the number of changed bit values and performs additional mutations if it is necessary. In this way, the mutation operator keeps the individuals correct.
- In the GAs for solving the USApHCP, USApHMP, CSApHCP, CSApHMP, CSAHLP the modified two-level mutation with frozen bits is used.
  Basic mutation rates are: 0.4/n, for the bit on the first position in the gene/the gens in the first segment, 0.1/n, for the bit on the second position in the gene/the first gene in the second segment, while the following bits in the gene are mutated with two times smaller rate than their predecessor bit: 0.05/n, 0.025/n, 0.0125/n,...
  Comparing to basic mutation rates, frozen bits (see Fig.8) are mutated with: 2.5 times higher rate, if a frozen bit is on the first position in the gene/in the first segment 1.5 times higher rate, otherwise

3.6 Other GA characteristics

- The initial population numbers 150 individuals
- 1/3 of the population is replaced in each generation, except the best 100 individuals that are directly passing in the next generations (elite individuals that preserve high fitted genes of the population)

- The infeasible individuals in the initial population are corrected to be feasible
- The applied genetic operators preserve the feasibility of individuals, so the infeasible individuals do not appear in the following generations
- If the individual with the same genetic code appears again in the population, its objective value is set to zero and the selection operator disables it to enter the next generation
- The appearance of individuals with the same objective value, but different genetic codes is limited to constant $N_{rv} = 40$
- This strategy helps in preserving the diversity of genetic material and in keeping the algorithm away from a local optima trap
- The running time of GA is improved by caching technique [28]. The main purpose of caching is to avoid the re-calculation of the same individuals objective value during the GA run

3.7 Review of computational results

The proposed GA approaches are tested on standard CAB (Civil Aeronautics Board) and AP (Australian Post) hub instances [4]. For the capacitated variants of hub location problems, two types of capacities and fixed costs on the nodes are assumed: tight (T) and loose (L), which gives four types of problems LL, LT, TL and TT for each problem size n.

Most of CAB instances ($n \leq 25$, $p \leq 4$) are easy to solve to optimality for all hub problems under consideration. For AP instances ($n \leq 200$, $p \leq 20$) optimal solutions are are mostly known for $n \leq 50$ problem dimensions. For some hub problems, no optimal solution is known even for smaller problem dimensions.

In all GA implementations two stopping criteria were used:

- The maximal number of generations $N_{gen} = 500$ for smaller, and $N_{gen} = 5000$ for larger problem instances,
- Algorithm also stops if the best individual or the best objective value remained unchanged through $N_{rep} = 200$ successive generations for smaller problem instances, and $N_{rep} = 2000$ for larger problem instances.

The detailed results of the proposed GAs and comparisons with the best-known heuristic and exact methods are presented in: [32], [33], [51], [52], [54]and [55]. On all the instances we considered, the combination of these two stopping criterions allowed GAs to converge to high-quality solutions. Only minor or no improvements in the quality of final solutions can be expected when prolonging the GA runs. From the results presented in the papers mentioned above, it is evident that the proposed GA methods quickly reach all previously known optimal solutions on CAB and smaller AP instances.

For all considered large AP instances, the GAs reach all previously known optimal solutions from the literature, with exception of two instances for the CSAHLP. For the CSApHMP and CSApHCP, the optimal solutions are obtained by using CPLEX in [51] and [55]. From the comparisons presented in the papers, it can be seen that proposed GAs reach or improve all best-known solutions obtained by existing heuristic methods, except for the CSAHLP. Unfortunately, most of the papers dealing with hub location problems don't provide results of the proposed heuristic methods on large-scale hub instances for which no optimal solution is known in advance. The maximal running time of the proposed GAs is usually few minutes. The longest GA run was 1.5 hours, in the case of largest AP instance for the UMApHCP for n=200, p=50.

3.8 Conclusion

The proposed GA methods provide good solutions in reasonable amount of time even for real-size hub instances. For some hub problems, GAs obtain solutions on some large AP instances for the first time. Although the optimality can not be proved, we believe that the obtained solutions are of high-quality.

Instead of ehaustive local search explorations, the GAs use representations of the solutions adopted to the problems' nature. Local search technique is employed only when it is necessary, so the running tim of the GAs is relatively short. This approach successfully leads the algorithm to the promising regions of the search space. The initial population is generated t be feasible and the implemented genetic operators preserve the feasibility, so that infeasible individuals do not appear. Mutation with "frozen bits" additionally keeps the diversibility of the genetic material. The computational results clearly indicate the advantages of the proposed GA implementations, especially for solving real-size hub instances. Therefore, the described GAs represent significant contribution to existing methods for solving hub location problems.

The future work will be realized in several directions:to parallelization of the GAs, execution on the multiprocessor, hybridization with exact methods and application of the similar evolutionary approach for solving some other hub location or combinatorial optimization problems.

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